**Problem 1 – Quantifiers**

Write the following statements as English sentences, then decide whether those statements are true if x and y can be any integers. When deciding if x and y can be any integers, prove your claim with a convincing argument.

1. **∀x∃y : x + y = 0**

English: *“For all elements x there exists an element y, such that x plus y is equal to zero.”*

Proof by cases:

Let us assume that x + y = 0, and therefore any element -x + y = 0:

|  |  |  |
| --- | --- | --- |
| **Case 1: x < 0** | **Case 2: x > 0** | **Case 3: x = 0** |
| x = -1 | x = 1 | x = 0 |
| y = -x = 1 | y = -x = -1 | y = x = 0 |
| **∴** -1 + 1 = 0 | **∴** 1 – 1 = 0 | **∴** 0 + 0 = 0 |

Therefore, it is **true**.

1. **∃y∀x : x + y = x**

English: *“There is an element y for all elements x, such that x plus y is equal to x.”*

Proof (Definition):

The sum of any number and 0, is equal to that number, such that

… which is true for all elements x given the identity property.

Therefore, it is **true**.

1. **∃x∀y : x + y = x**

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

Therefore, it is **false**.

**Problem 2 – Growth of Functions**

Organize the following functions into six columns. Items in the same column should have the same asymptotic growth rates (they are big-O and big-Θ of each other). If a column is to the left of another column, all its growth rates should be slower than those of the column to its right.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **O(1)** | **O(Log n)** | **O(n)** | **O(n log n)** | **O(n2)** | **O(n3)** | **O(2N)** | **O(N!)** |
| 10000  100 |  | 3n | n log2 n  n log3 n | n2  5n2+3 |  | 2^n | N! |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **O(1)** | **O(n)** | **O(n log n)** | **O(n2)** | **O(2N)** | **O(N!)** |
| 10000  100 | 3n  100n | n log2 n  n log3 n | n2  5n2+3 | 2n | n! |

**Problem 3 – Function Growth Language**

Match the following English explanations to the best corresponding Big-O function by drawing a line from the left to the right.

|  |  |  |
| --- | --- | --- |
| 1 | Constant Time | O(1) |
| 2 | Logarithmic Time | O(log n) |
| 3 | Linear Time | O(n) |
| 4 | Quadratic Time | O(n2) |
| 5 | Cubic Time | O(n3) |
| 6 | Exponential Time | O(2n) |
| 7 | Factorial Time | O(n!) |

**Problem 4 – Big-O**

1. Using the definition of big-O, show 100n + 5 = O(2n).

|  |  |
| --- | --- |
|  | When n=1, k=1, using our equation  **∴** The function is O(2n) because when the constant k=1 and constant c = 105/2, the definition:  … for all positive numbers greater than or equal to zero. |

1. Using the definition of big-O, show n 3 + n 2 + n + 100 = O(n 3 ).